

Quark mass dependence of light resonances and phase shifts in elastic $\pi\pi$ and πK scattering

Jenifer Nebreda and J. R. Peláez
Universidad Complutense de Madrid

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Williamsburg, Virginia

Outline

1 Motivation

2 Standard ChPT

- $\pi\pi$ scattering in SU(2) standard ChPT
- SU(2) standard ChPT phase shifts dependence on \hat{m}

3 Unitarized ChPT

- Inverse Amplitude Method
- SU(2) Unitarized ChPT phase shifts dependence on \hat{m}
- Scalar and vector mesons dependence on quark masses

4 Summary

Motivation

Lattice: rigorous QCD results with quarks and gluons.

Growing interest in scattering and scalar sector.

Caveat: small, realistic quark masses are difficult to implement.

ChPT: QCD dependence on quark masses as an expansion.

We can compare:

Lattice multi-hadron states calculations \rightarrow phase shifts and scattering lengths	vs.	standard ChPT (model independent) or UChPT (to go higher in \sqrt{s})
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Lattice spectrum calculations \rightarrow masses	vs.	UChPT
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Standard Chiral Perturbation Theory

Chiral Perturbation Theory

Weinberg, Gasser & Leutwyler

Low energy effective theory of QCD with:

- DOF: Pseudo-Goldstone Bosons of the spontaneous chiral symmetry breaking

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

$N_f=2 \rightarrow \pi$'s

$N_f=3 \rightarrow \pi$'s, K 's and η

- expansion in masses and momenta

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

- parameters: Low Energy Constants (LECs)

$N_f=2 \rightarrow 4 l$'s (one loop) and 7 r 's (two loops)

$N_f=3 \rightarrow 8 L$'s (one loop)

$\pi\pi$ scattering in SU(2) standard ChPT:

- Already calculated to 1 and 2 loops*, we study the phases dependence on $\hat{m} = \frac{m_u+m_d}{2}$.
- Advantages:
 - SISTEMATIC EXPANSION, MODEL INDEPENDENT
 - some lattice groups already giving results for $I=2$ phases and scattering lenghts**
- Limitations:
 - only low energy region
 - no resonances.

*J. Bijnens, G. Colangelo, G. Ecker, J. Gasser and M. E. Sainio, Phys. Lett. B **374**, 210 (1996)

** K. Sasaki and N. Ishizuka, Phys. Rev. D **78**, 014511 (2008)

Standard $SU(2)$ ChPT amplitudes with LECs from

G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B **603**, 125 (2001)

$O(p^{-4})$ LECs ($\times 10^{-3}$)	$O(p^{-6})$ LECs ($\times 10^{-4}$)
I_1^r	-3.98 ± 0.62
I_2^r	1.89 ± 0.23
I_3^r	0.82 ± -3.80
I_4^r	6.17 ± 1.39
	r_1^r
	r_2^r
	r_3^r
	r_4^r
	r_5^r
	r_6^r

Statistical error, not systematic

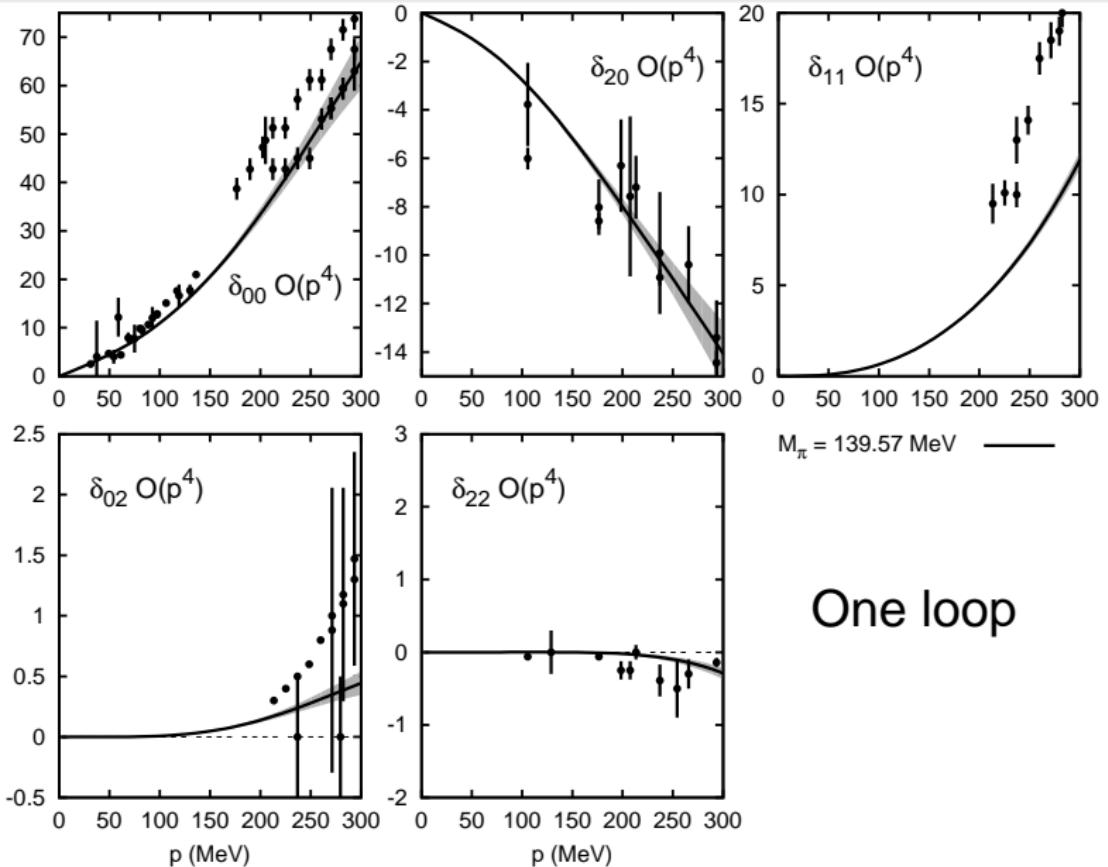
Change \hat{m} \Rightarrow change on $M_\pi^2 = 2\hat{m}B_0$ \Rightarrow change on f_π

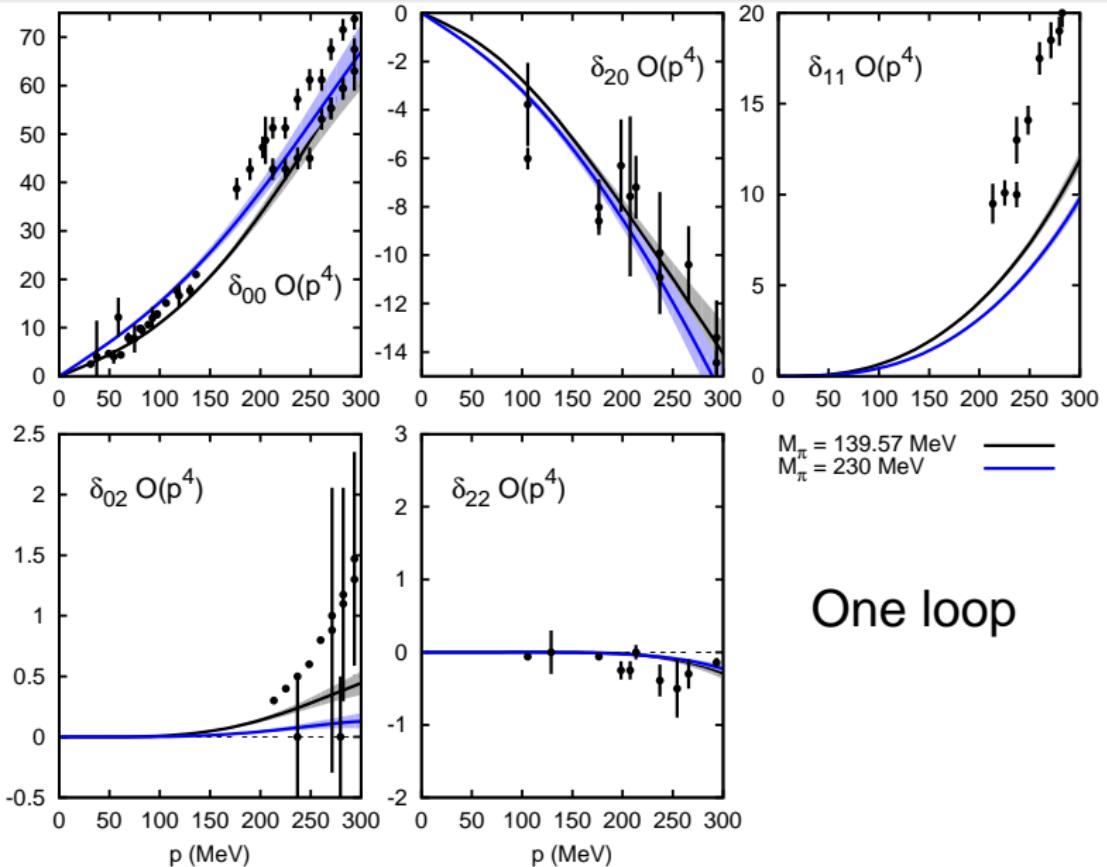
(one more $O(p^6)$ parameter: $r_f^r \approx 0 \pm 1.2 \times 10^{-4}$)

Uncertainties in phase shifts: **Montecarlo Gaussian Sampling.**

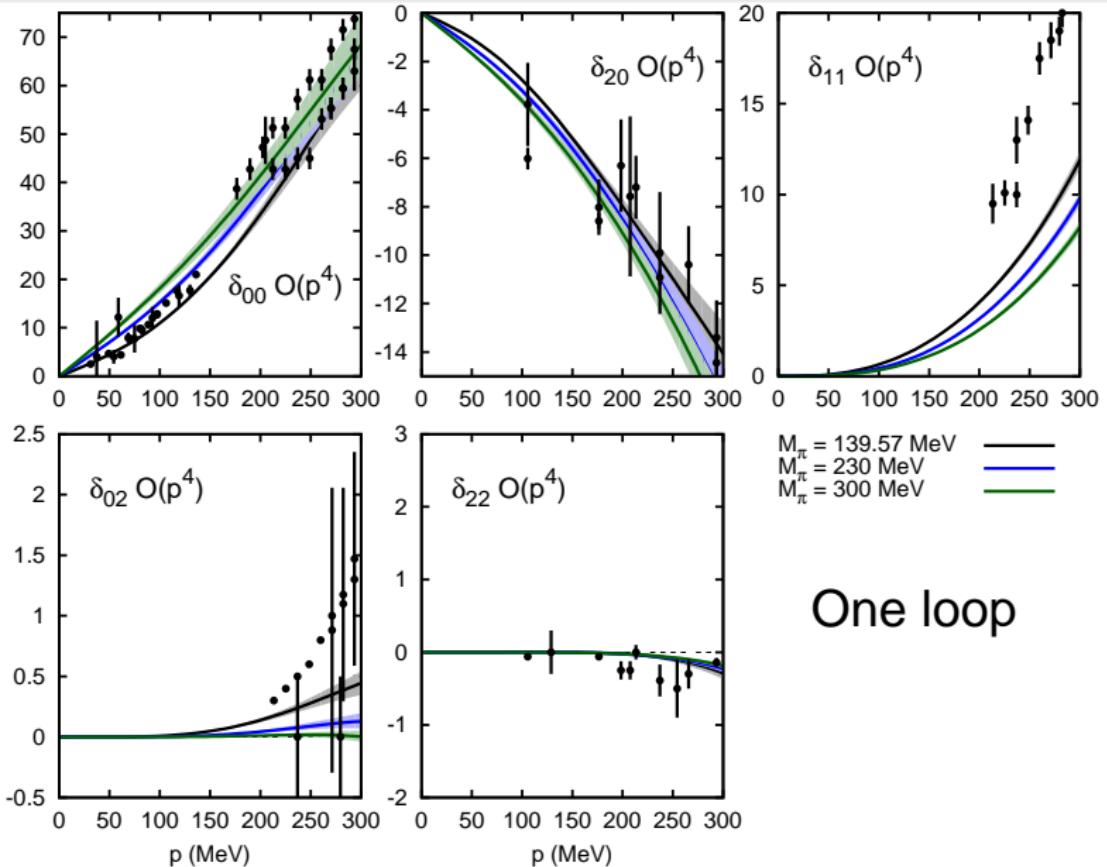
Phase shifts vs. Momentum, increasing M_π

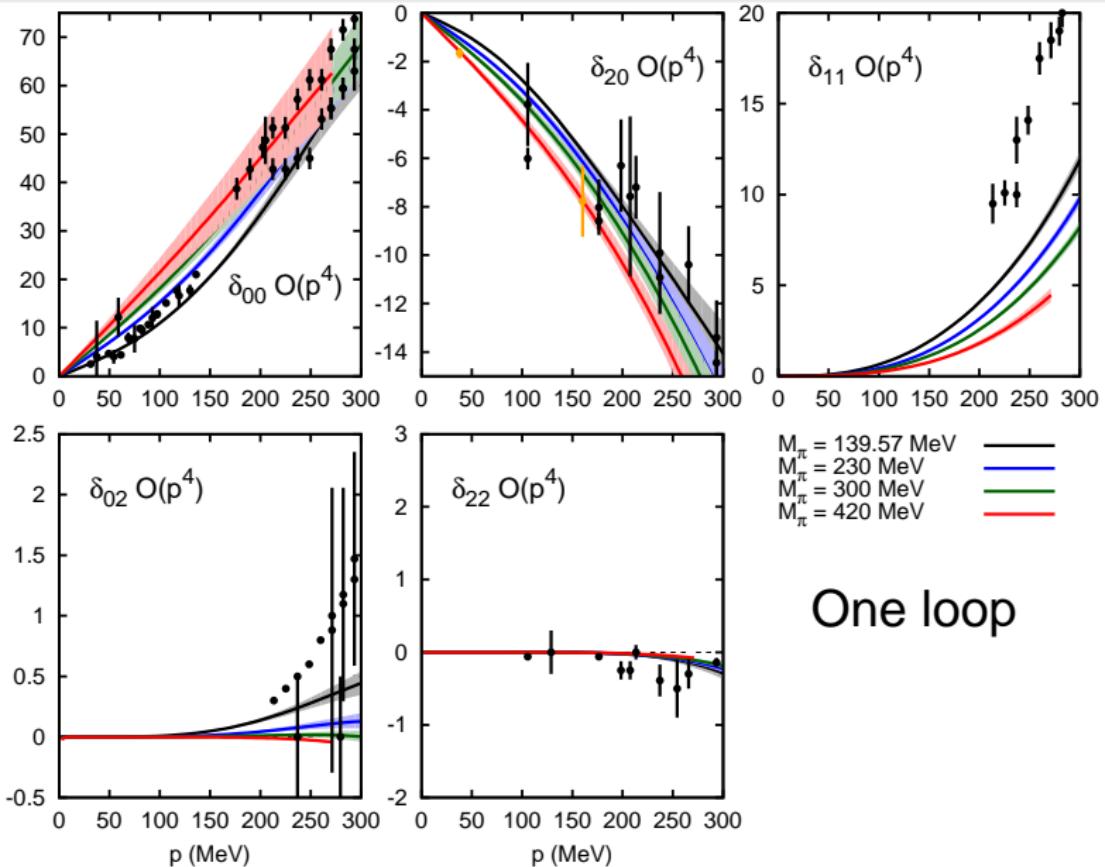
Phases vs. energy $\rightarrow \hat{m}$ dependence from the threshold's shift.
Better to plot phases vs. momentum.



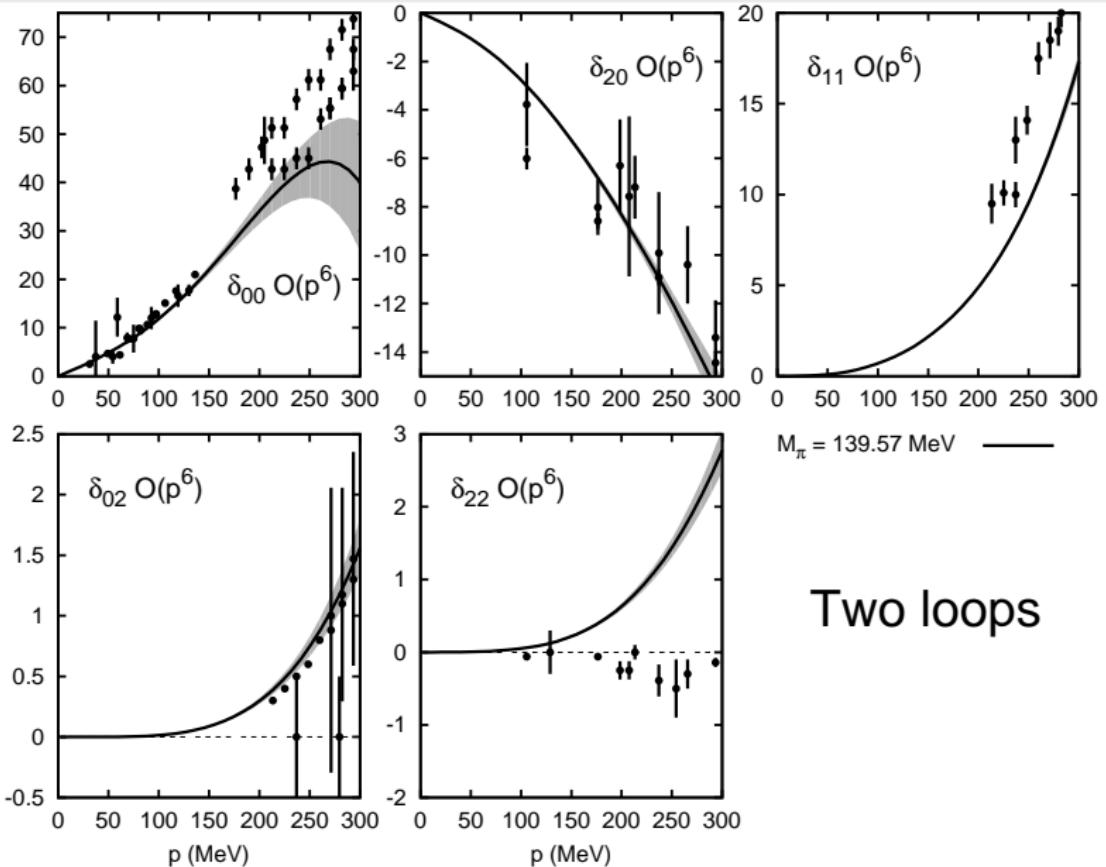


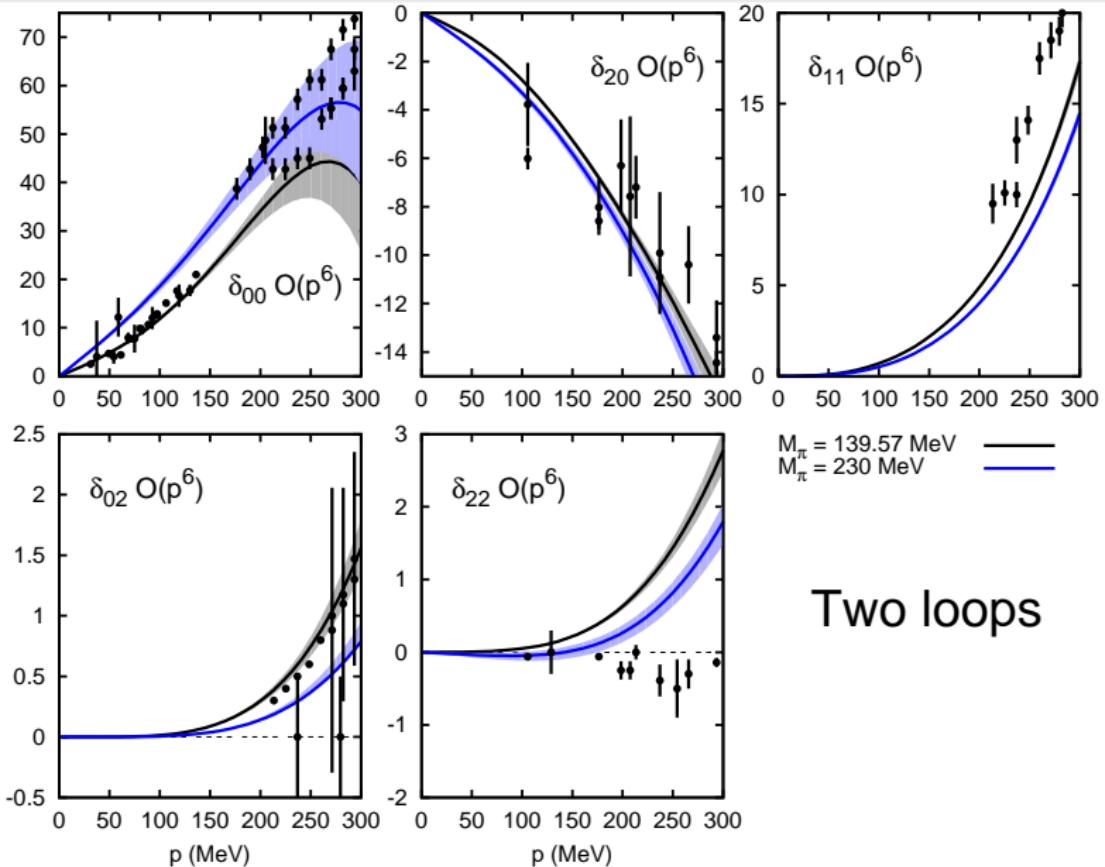
One loop



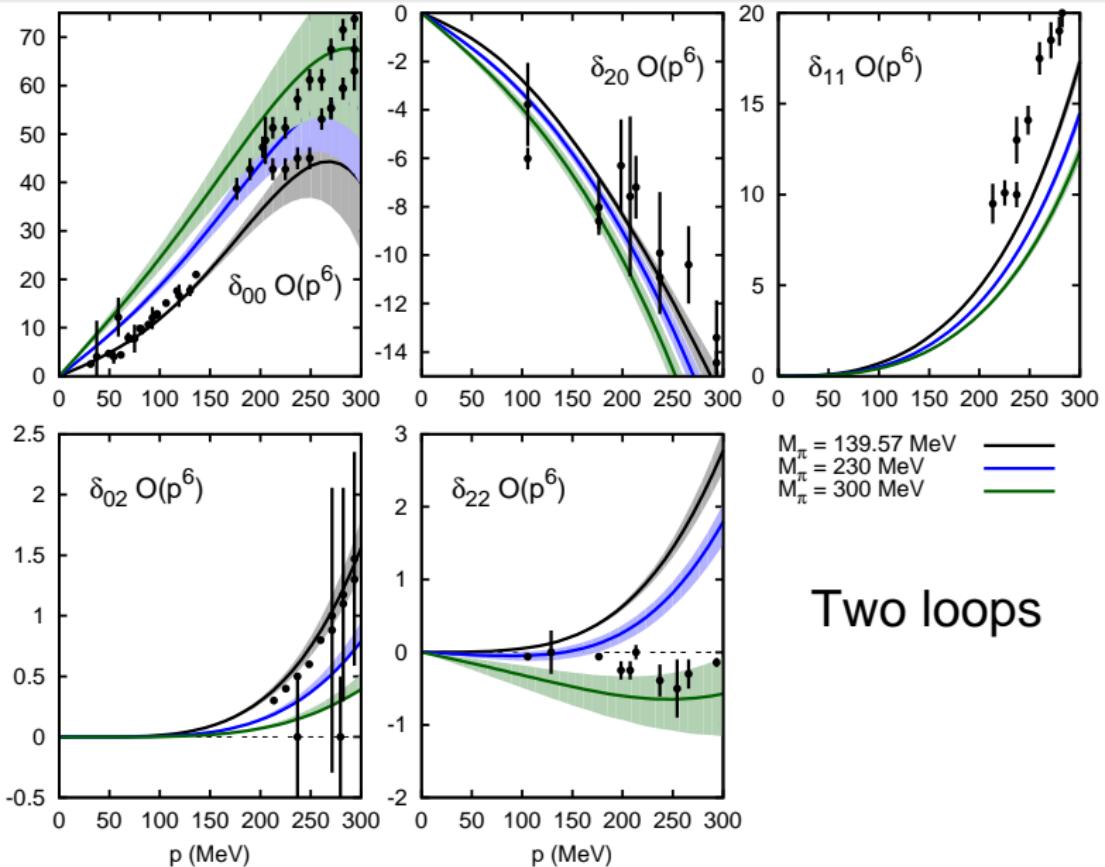


* K. Sasaki and N. Ishizuka, Phys. Rev. D 78, 014511 (2008)

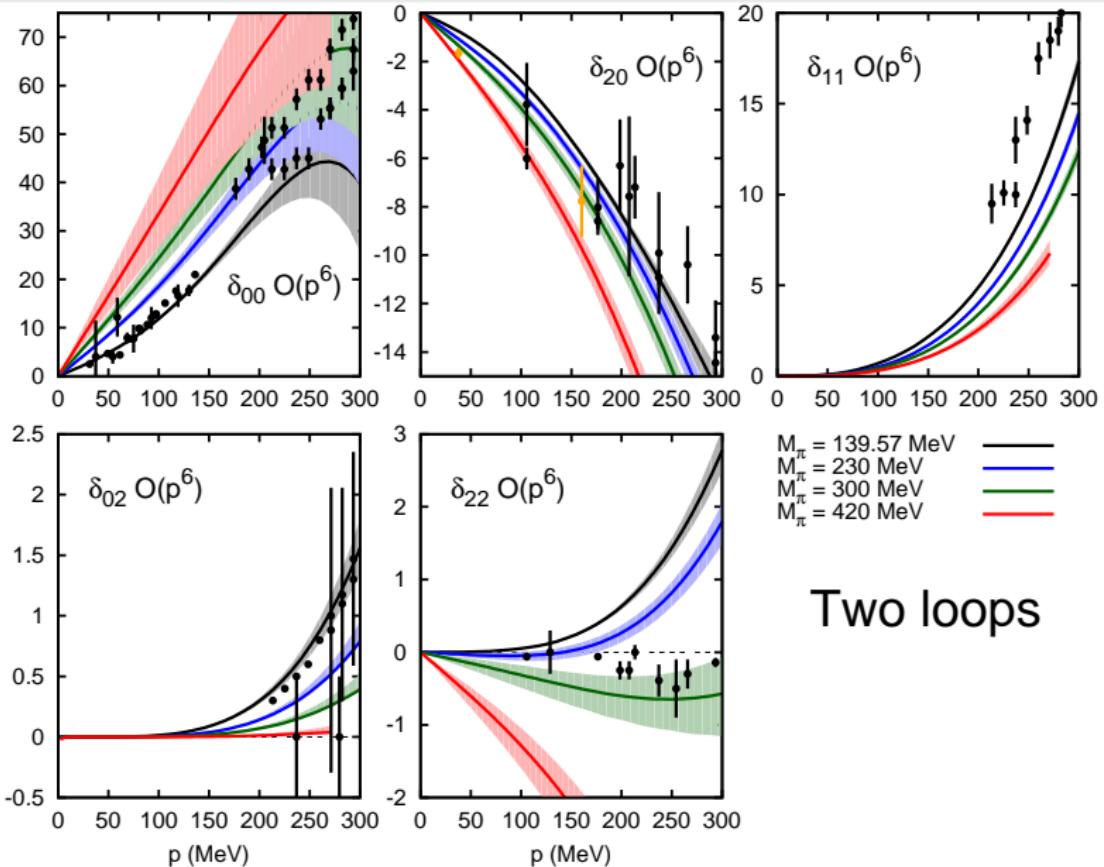




Two loops



Two loops



Two loops

* K. Sasaki and N. Ishizuka, Phys. Rev. D 78, 014511 (2008)

Unitarized ChPT

Inverse Amplitude Method Truong, Dobado, Herrero, Peláez

Elastic IAM partial waves satisfy exact unitarity

$$\mathbf{S}\mathbf{S}^\dagger = 1 \Rightarrow \text{Im } t^{-1} = -\sigma$$

$O(p^4)$ IAM partial waves:

$$t(s) \simeq \frac{t_2^2(s)}{t_2(s) - t_4(s)}$$

It is derived from a dispersion relation:

- exact on the elastic right cut,
- left cut and subtraction constants approximated within NLO ChPT,
- fully renormalized,
- no spurious parameters.

Inverse Amplitude Method Truong, Dobado, Herrero, Peláez

Elastic IAM partial waves satisfy exact unitarity

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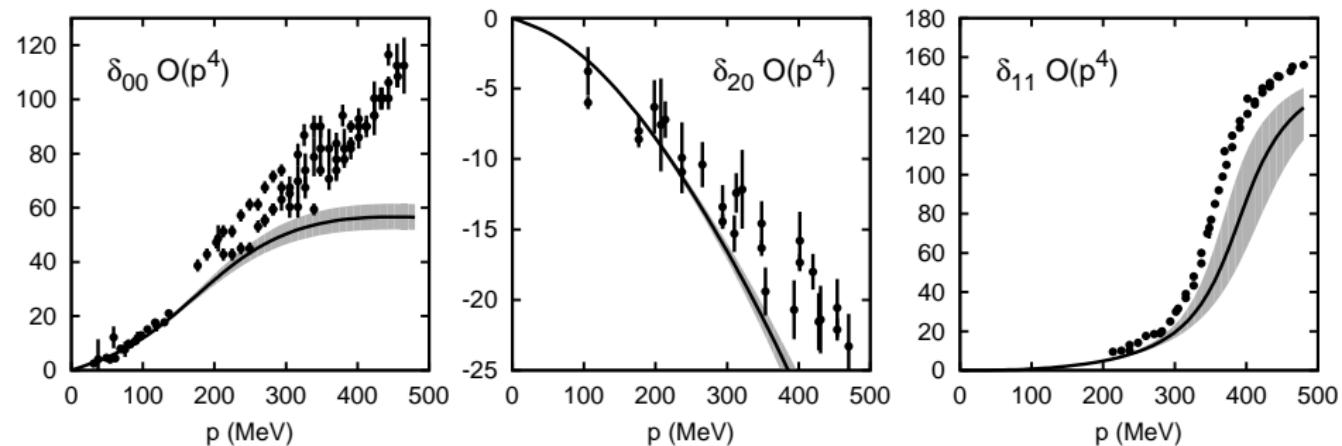
$O(p^6)$ IAM partial waves:

$$t(s) \simeq \frac{t_2^2(s)}{t_2(s) - t_4(s) + \frac{t_4^2}{t_2} - t_6}$$

It is derived from a dispersion relation:

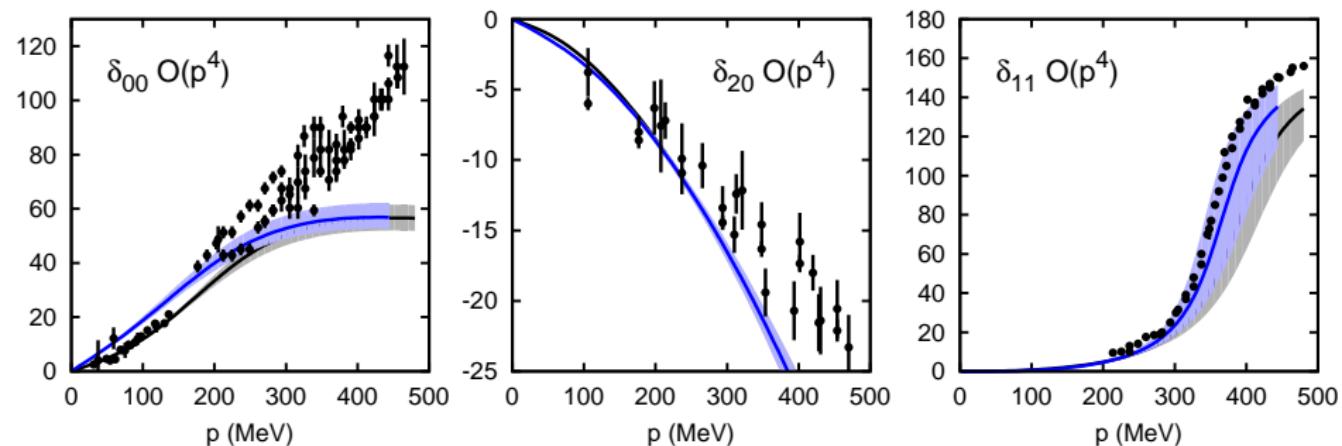
- exact on the elastic right cut,
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SU(2) Unitarized ChPT phase shifts vs. Momentum



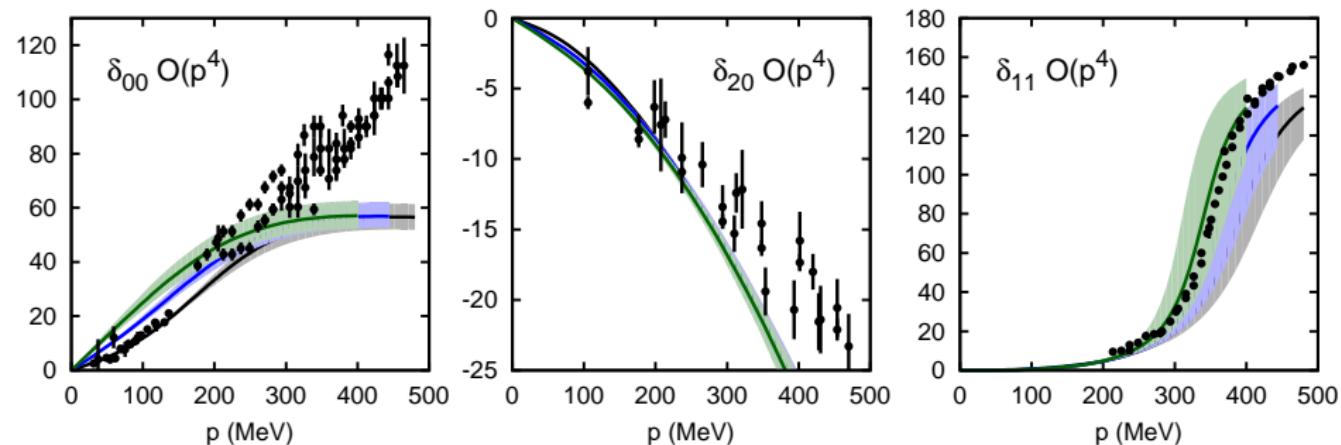
$M_\pi = 139.57$ MeV —————

One loop



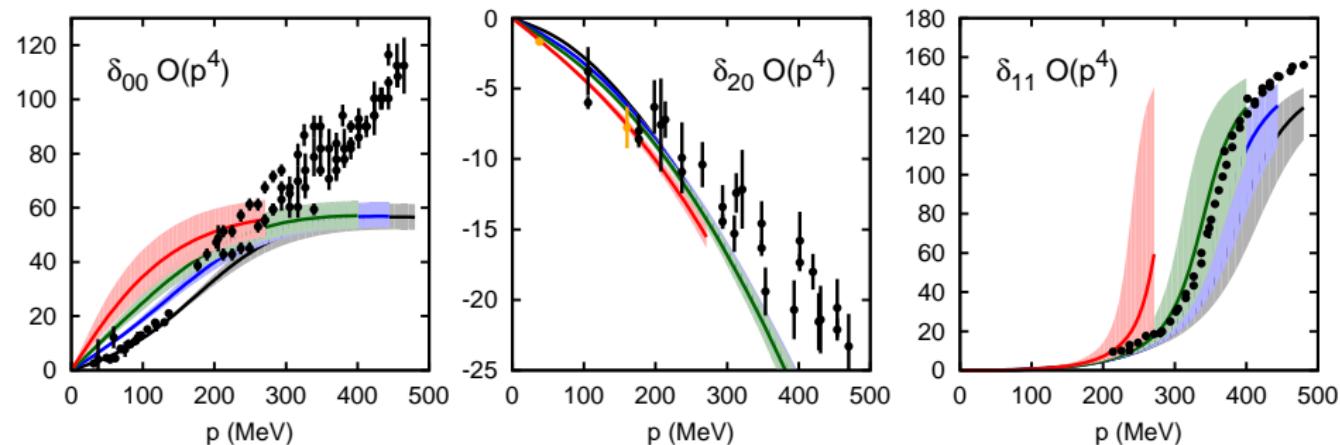
$M_\pi = 139.57$ MeV —
 $M_\pi = 230$ MeV —

One loop



$M_\pi = 139.57 \text{ MeV}$ —
 $M_\pi = 230 \text{ MeV}$ —
 $M_\pi = 300 \text{ MeV}$ —

One loop



$M_\pi = 139.57$ MeV
 $M_\pi = 230$ MeV
 $M_\pi = 300$ MeV
 $M_\pi = 420$ MeV

One loop

* K. Sasaki and N. Ishizuka, Phys. Rev. D 78, 014511 (2008)

Scalar and vector mesons dependence on quark masses

Quark mass dependence

Generalization to $SU(3)$ of previous work on $SU(2)^*$.

Elastic channels:

- $\pi\pi \rightarrow \pi\pi$: resonances ρ and σ (comparison to $SU(2)$ results)
- $\pi K \rightarrow \pi K$: resonances $K^*(892)$ and κ .

Change of $\hat{m} = \frac{m_u + m_d}{2}$ and $m_s \Rightarrow$

change of M_π^2 , M_K^2 , M_η^2 , f_π , f_K , f_η .

Applicability in $SU(3)$: $0 < M_\pi \lesssim 400$ MeV $\Rightarrow M_K \lesssim 600$ MeV
(Being optimistic!)

* C. Hanhart, J.R. Pelaez and G. Rios, Phys. Rev. Lett. **100**, 152001 (2008)

Our approach

$SU(3)$ analysis has many more chiral parameters than $SU(2)$ and worse determined by scattering data.

Thus we:

- fit to experimental data on the elastic region to fix L_1 , L_2 and L_3 ,
- fit to lattice results on NGB masses, decay constants and scattering lengths to fix better L_4 , L_5 , L_6 and L_8 .

Low energy constants from fits

$\mathcal{O}(p^4)$ parameters ($\times 10^3$)

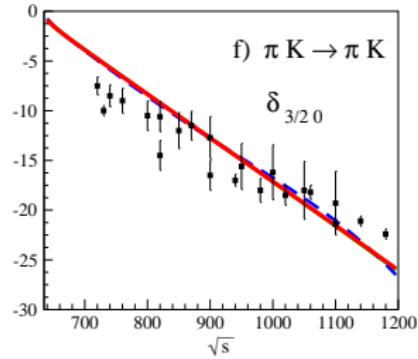
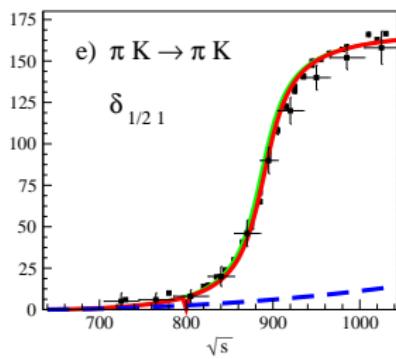
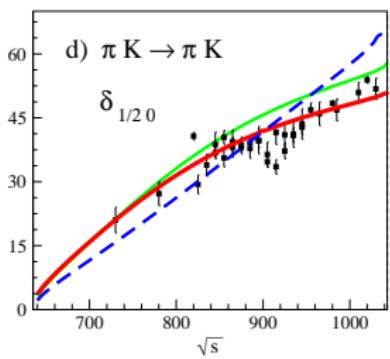
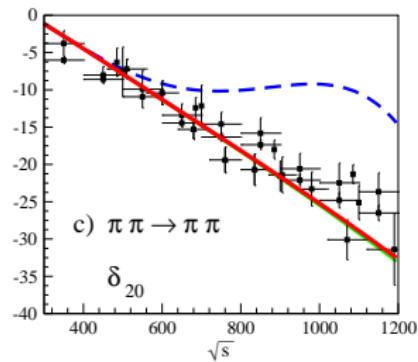
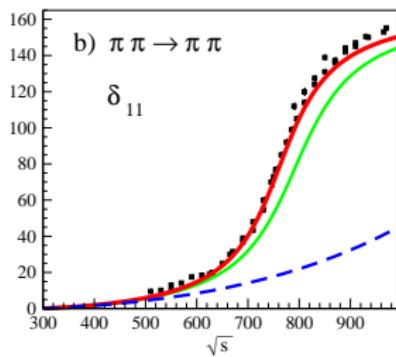
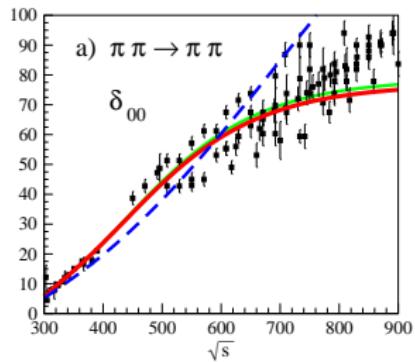
LECs	$O(p^6)^*$	$O(p^4)^*$	**	Fit I	Fit II
L_1^r	0.53 ± 0.25	0.46	1.05 ± 0.12	1.10	0.74
L_2^r	0.71 ± 0.27	1.49	1.32 ± 0.03	1.11	1.04
L_3	-2.72 ± 1.12	-3.18	-4.53 ± 0.14	-4.03	-3.12
L_4^r	0 (fixed)	0 (fixed)	0.53 ± 0.39	-0.06	0.00
L_5^r	0.91 ± 0.15	1.46	3.19 ± 2.40	1.34	1.26
L_6^r	0 (fixed)	0 (fixed)	-	0.15	-0.01
L_7	-0.32 ± 0.15	-0.49	-	-0.43	-0.49
L_8^r	0.62 ± 0.20	1.00	-	0.94	1.06
$L_8^r + 2L_6^r$	0.62 ± 0.20	1.00	3.66 ± 1.52	1.24	1.04
$2L_1 - L_2$	0.35 ± 0.57	-0.57	0.78 ± 0.24	1.09	0.44

- Simultaneous description of low energy and resonances,
- fully renormalized, LECs compatible with standard ChPT.

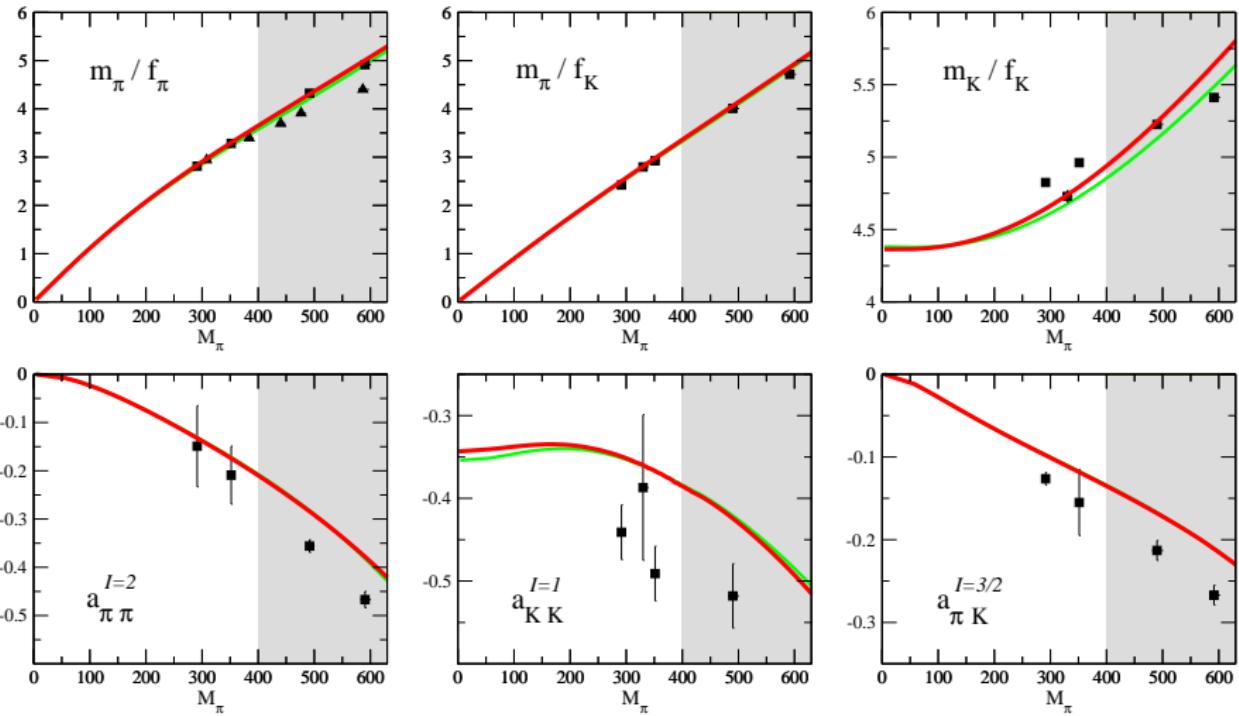
* G. Amoros et al., Nucl. Phys. B **602**, 87 (2001)

** P. Buettiker et al., Eur. Phys. J. C **33**, 409 (2004); S. Descotes-Genon et al., Eur. Phys. J. C **48**, 553 (2006).

IAM fits to elastic scattering phase shift data



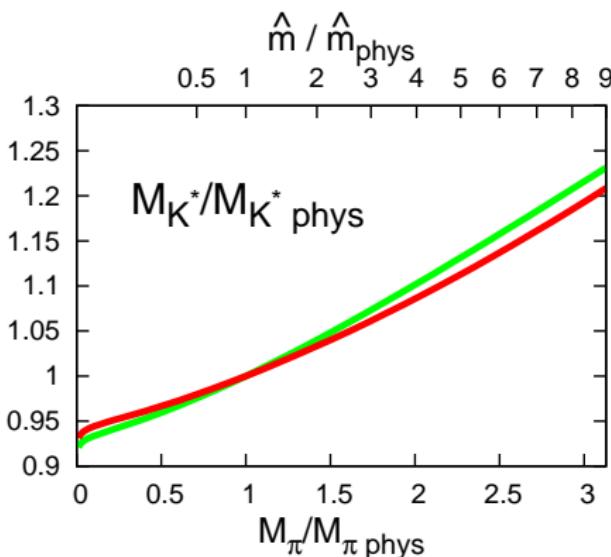
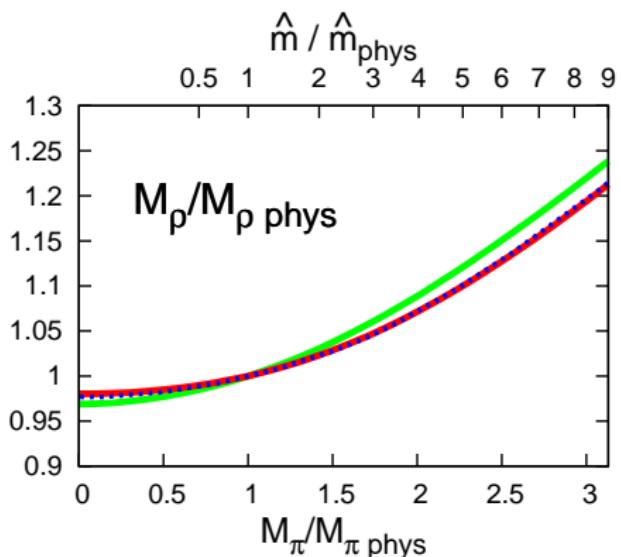
IAM fits to lattice on $\frac{m_\pi}{f_\pi}$, $\frac{m_\pi}{f_K}$, $\frac{m_K}{f_K}$ and scattering lengths



\hat{m} dependence of σ , ρ , κ and $K^*(892)$

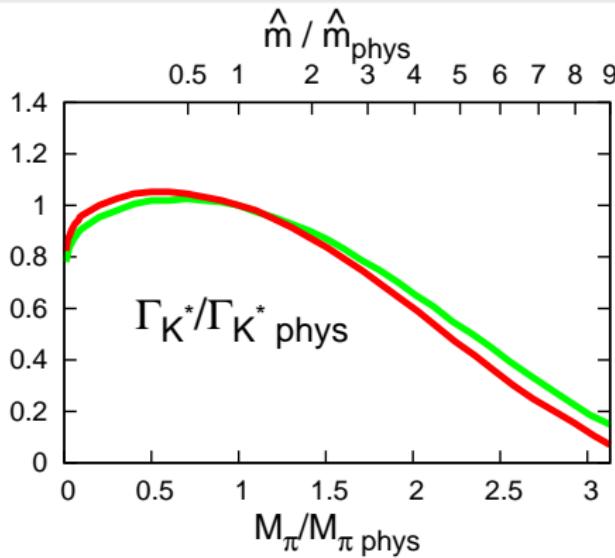
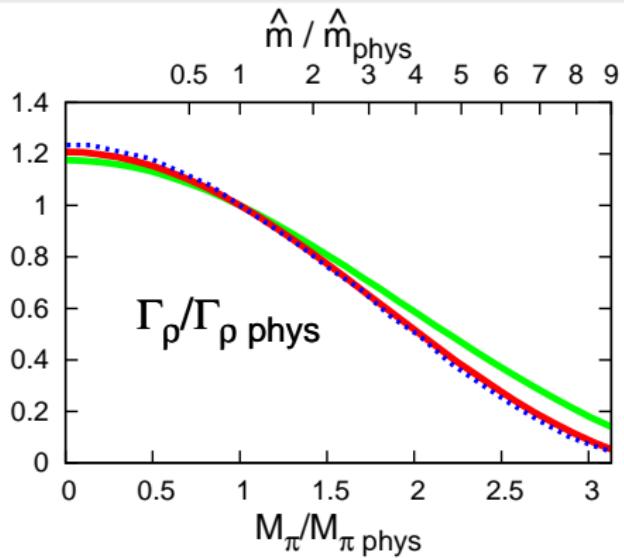
Light vector mesons: ρ and $K^*(892)$

\hat{m} dependence - Light vector mesons - Mass

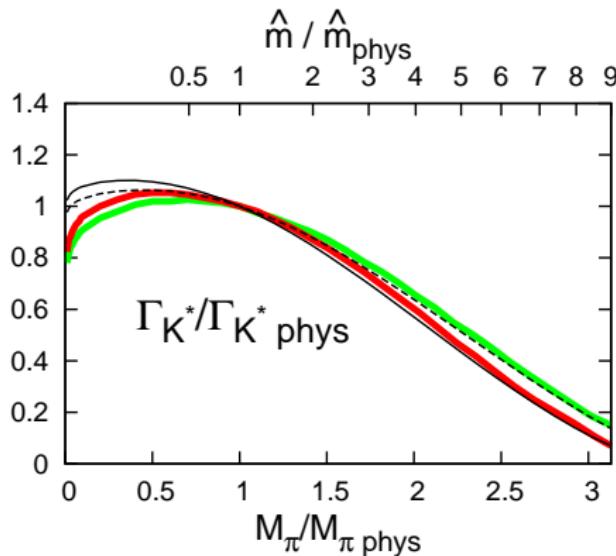
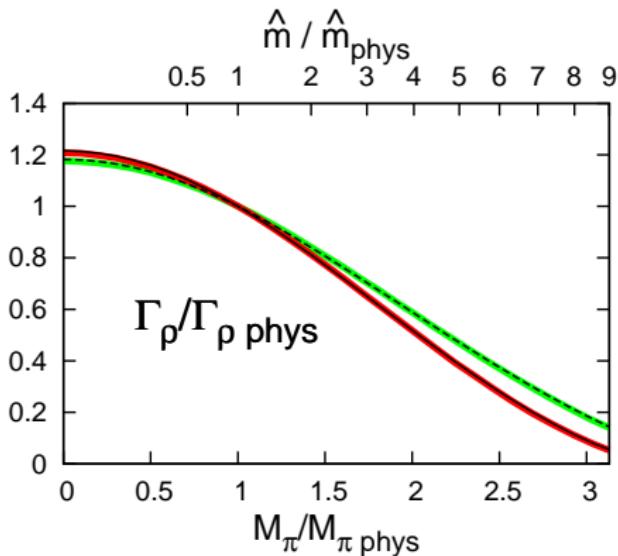


- Both masses increase slower than M_π
- Agreement with SU(2) analysis (blue line)

\hat{m} dependence - Light vector mesons - Width



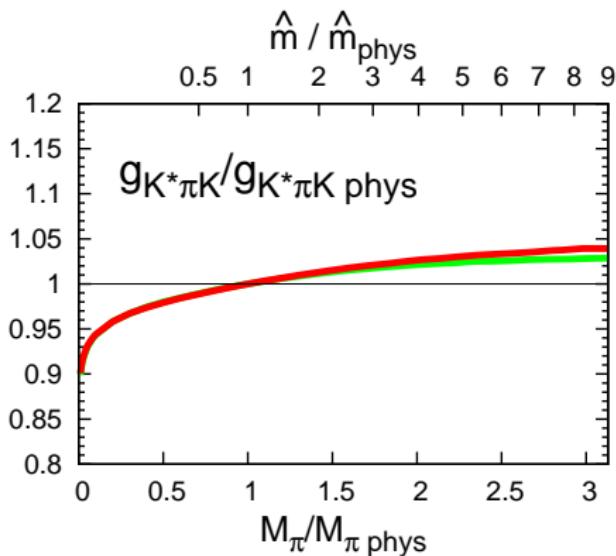
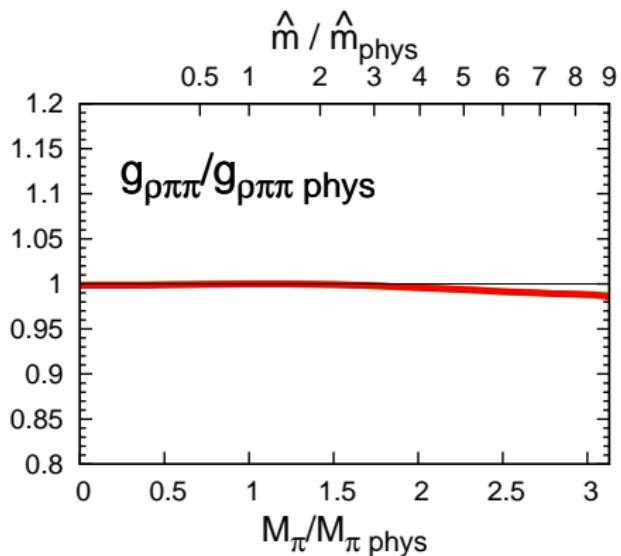
\hat{m} dependence - Light vector mesons - Width



- Width decrease in accordance with phase space reduction:

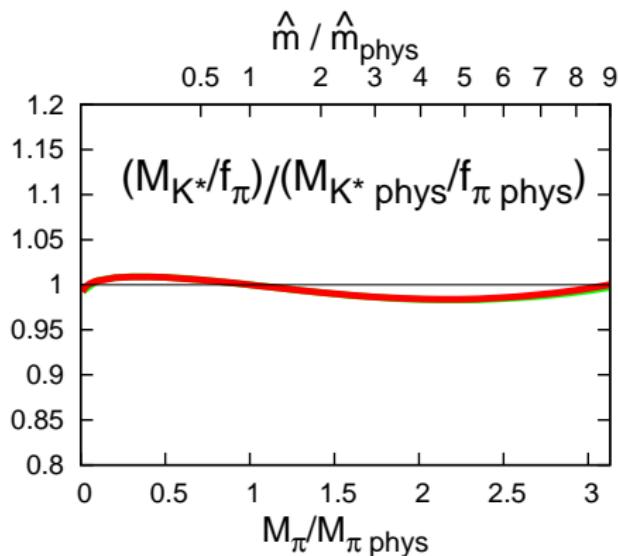
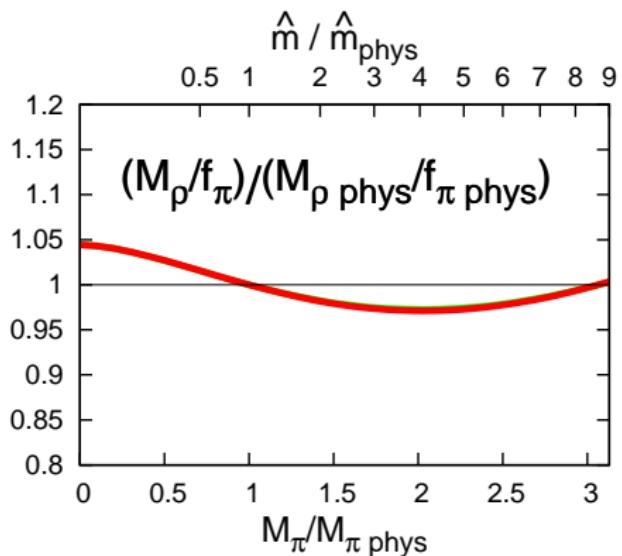
$$\Gamma_V = g^2 \frac{1}{8\pi} \frac{|\mathbf{p}|^3}{M_V^2} \quad (\text{black lines})$$

\hat{m} dependence - Light vector mesons - Coupling



- Coupling to two mesons independent of \hat{m} (assumption in some lattice works)

\hat{m} dependence - Light vector mesons - KSFR

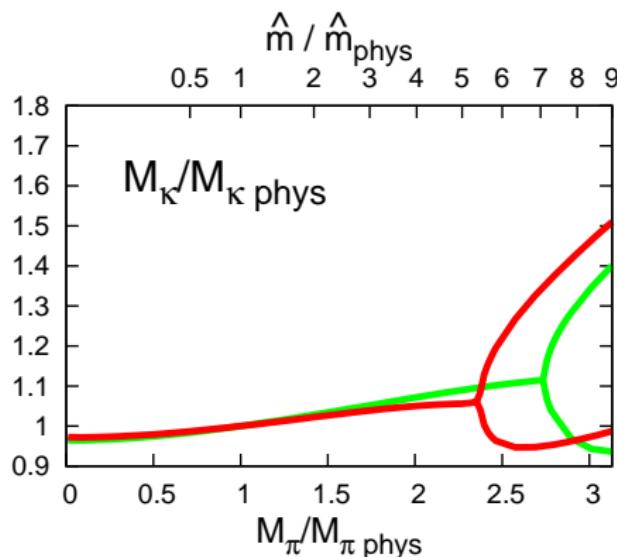
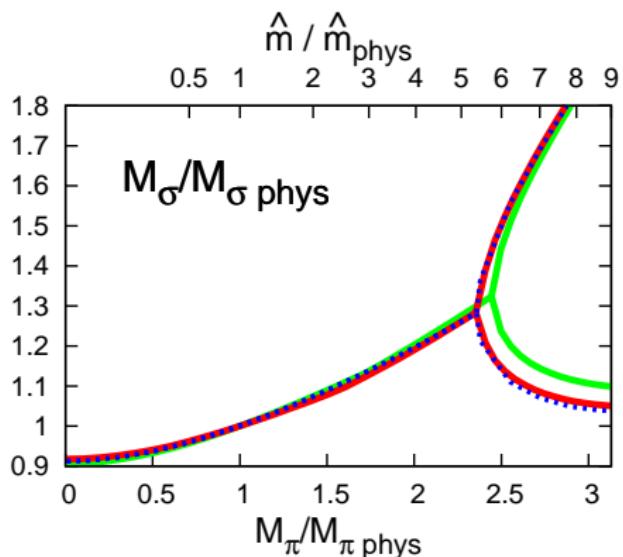


- Fulfill the KSFR relation for different \hat{m} :

$$g \simeq M_V/2\sqrt{2}f_\pi$$

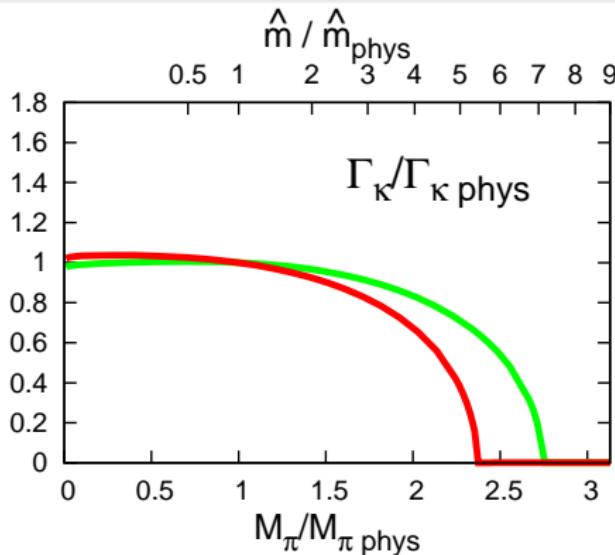
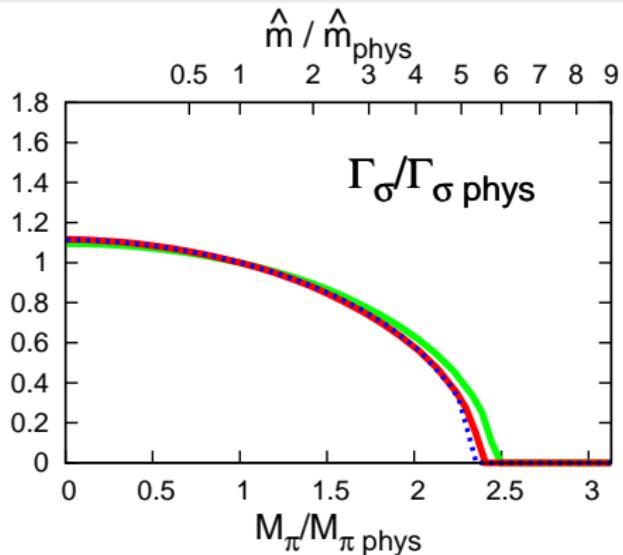
Light scalar mesons: σ and κ

\hat{m} dependence - Light scalar mesons - Mass

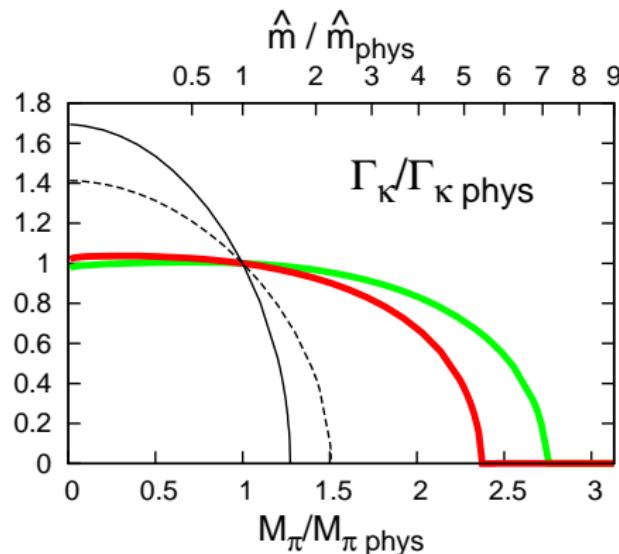
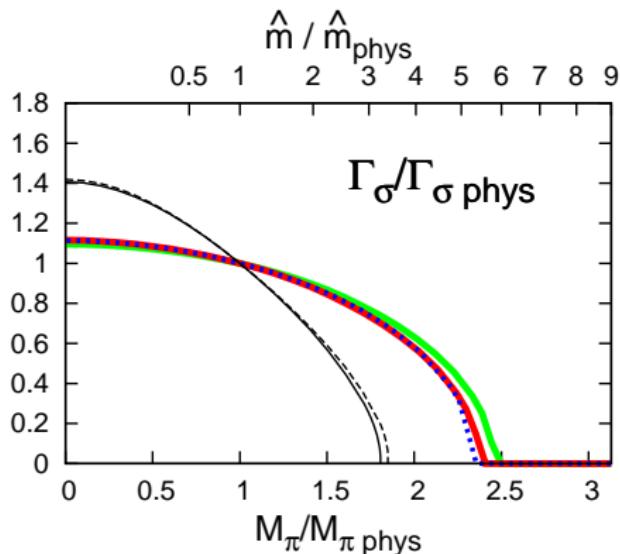


- Mass split into two branches
- Agreement with SU(2) analysis

\hat{m} dependence - Light scalar mesons - Width



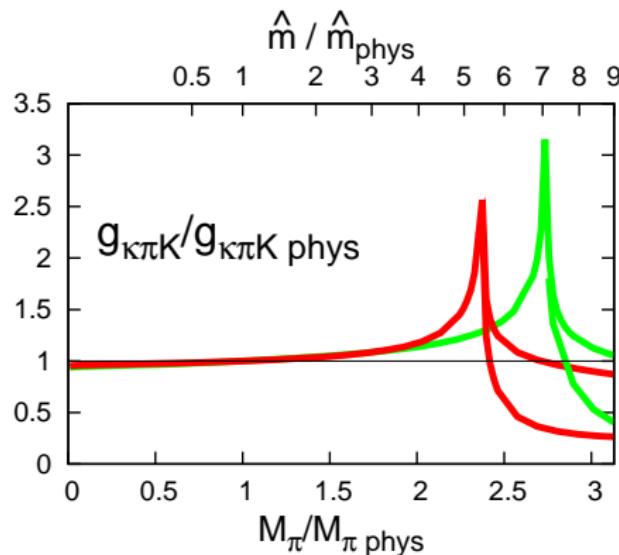
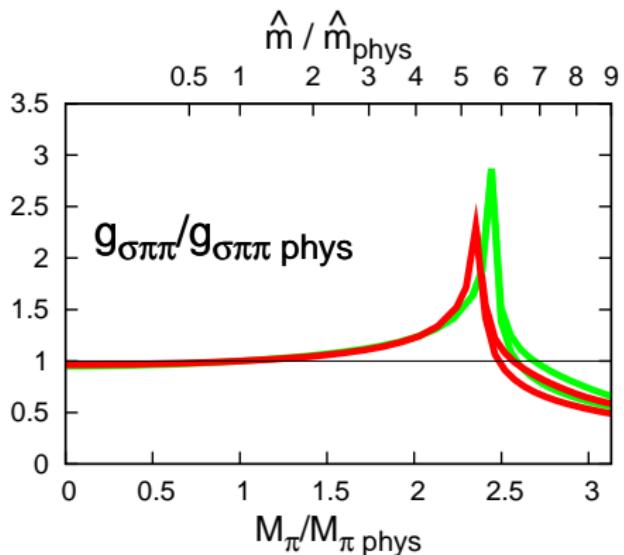
\hat{m} dependence - Light scalar mesons - Width



- Width decrease not explained by phase space reduction:

$$\Gamma_S = g^2 \frac{1}{8\pi} \frac{|\mathbf{p}|}{M_S^2}$$

\hat{m} dependence - Light scalar mesons - Coupling

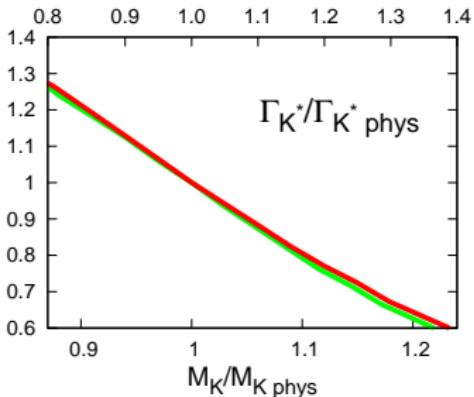
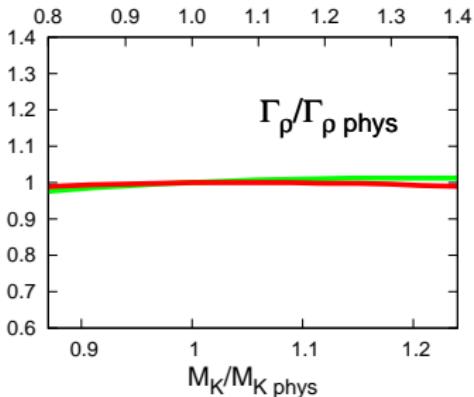
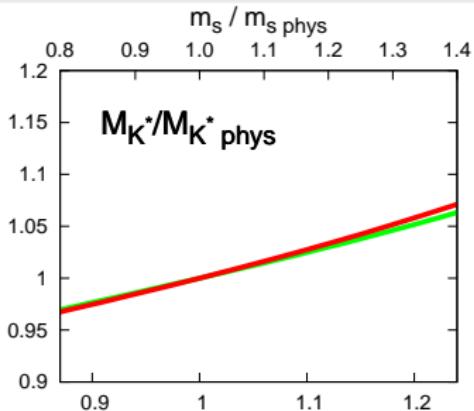
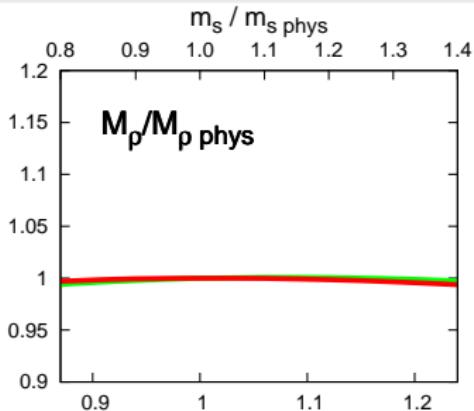


- Strong \hat{m} dependence of coupling to two mesons

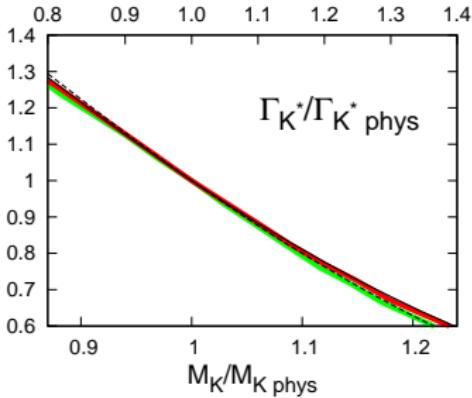
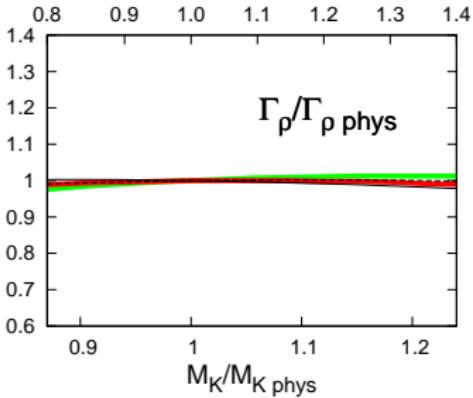
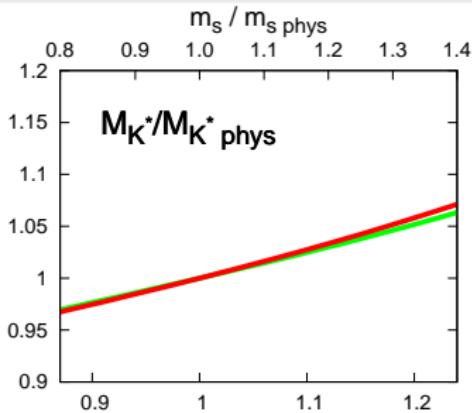
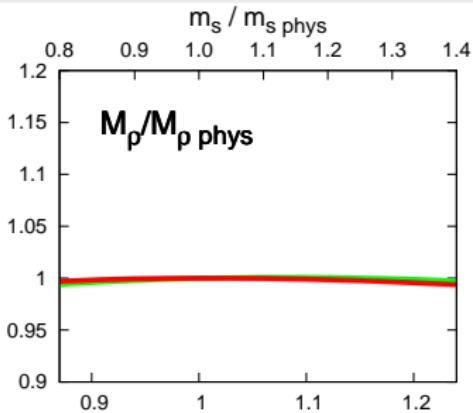
m_s dependence of σ , ρ , κ and $K^*(892)$

Light vector mesons: ρ and $K^*(892)$

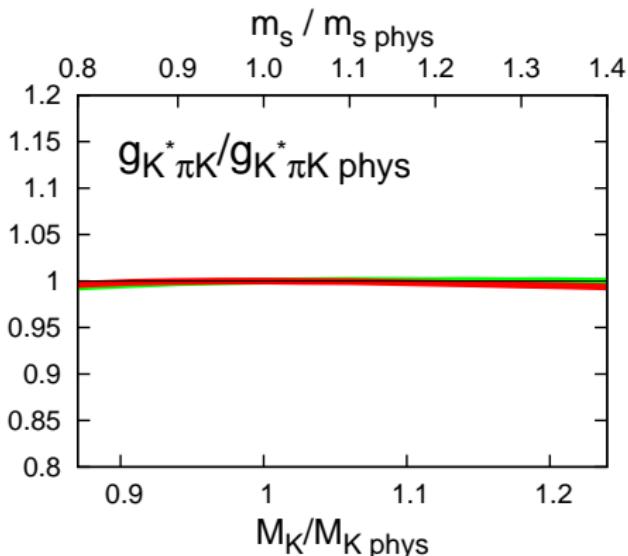
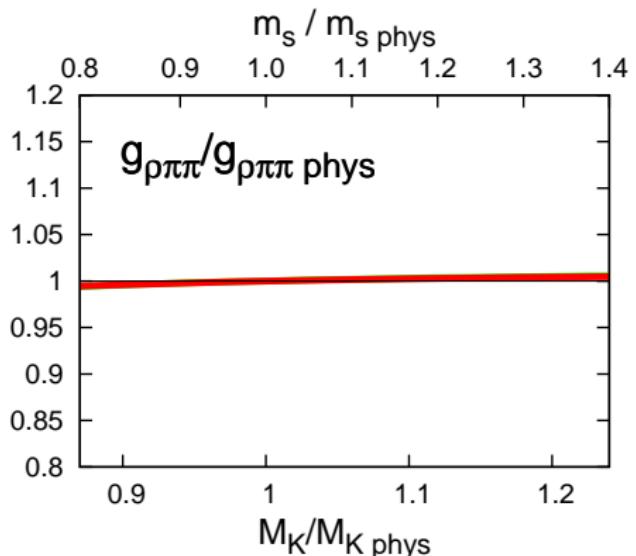
m_s dependence - Light vector mesons - Mass & Width



m_s dependence - Light vector mesons - Mass & Width

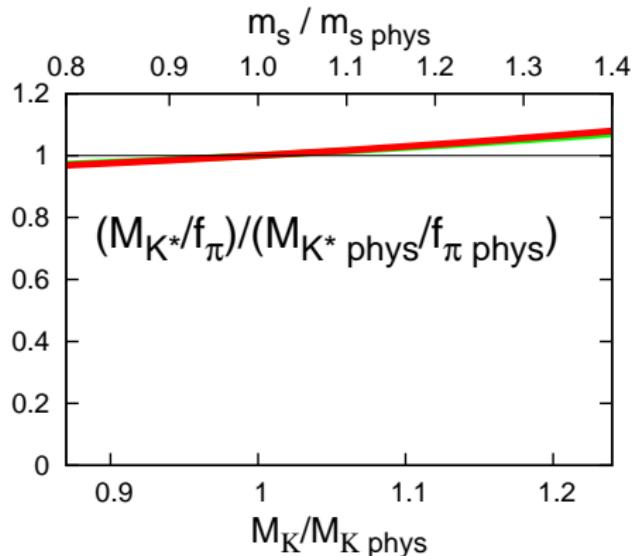
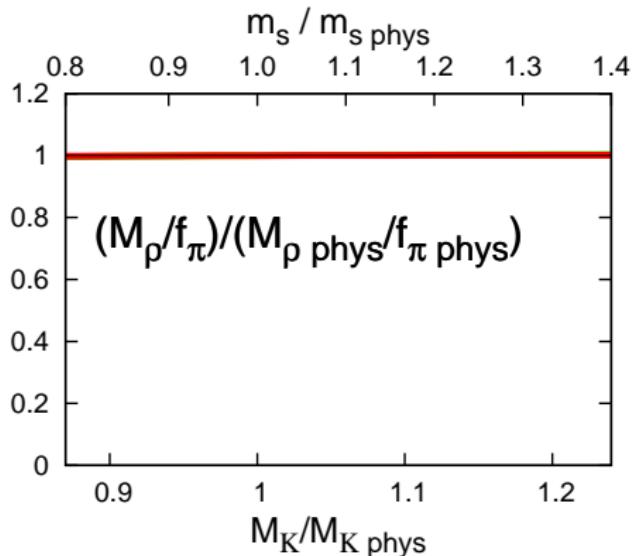


m_s dependence - Light vector mesons - Coupling



■ Coupling to two mesons constant

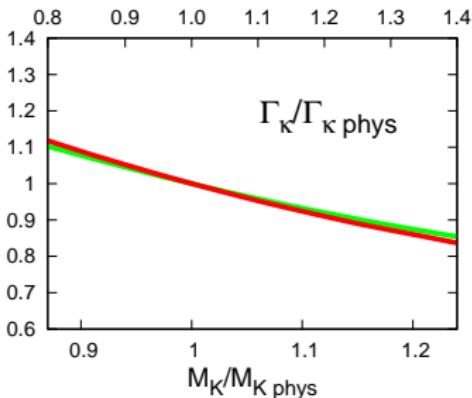
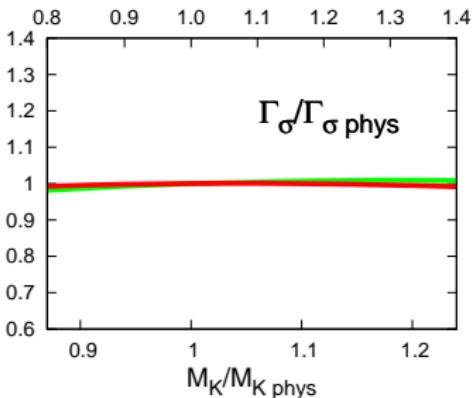
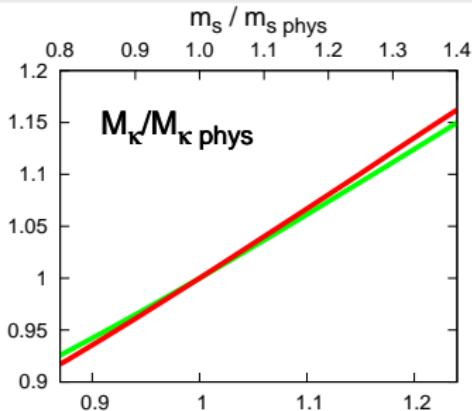
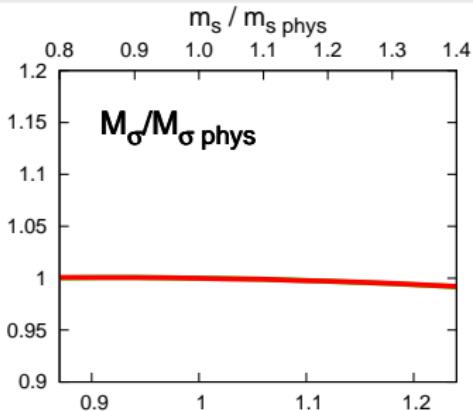
m_s dependence - Light vector mesons - KSFR



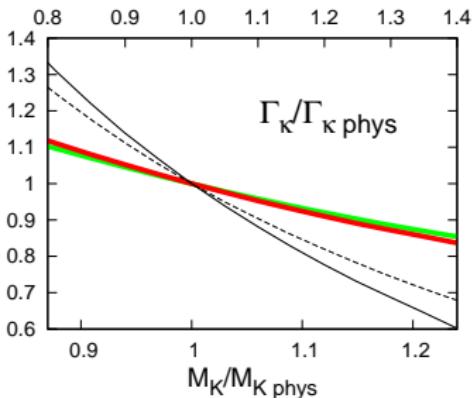
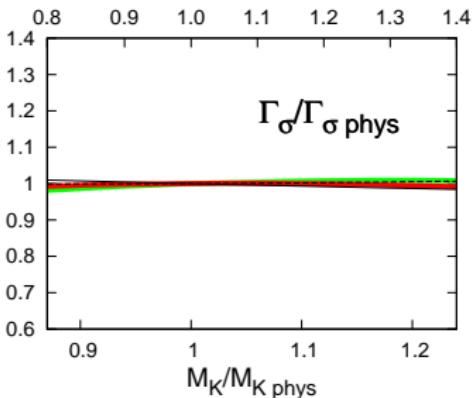
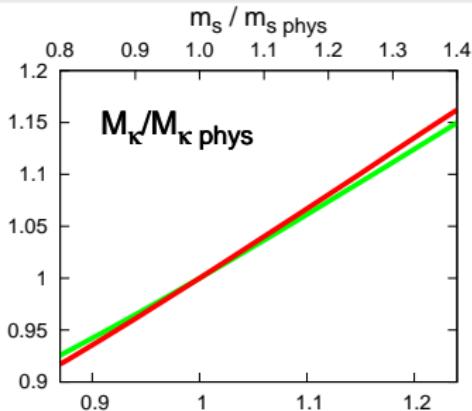
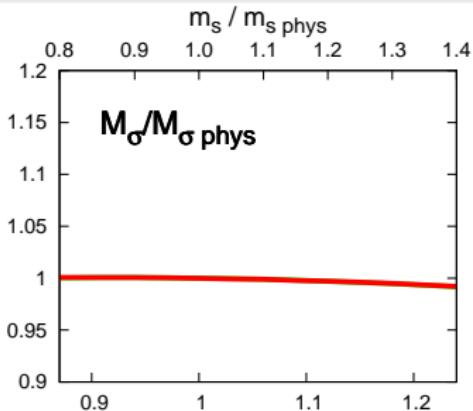
■ KSFR relation well satisfied for different m_s

Light scalar mesons: σ and κ

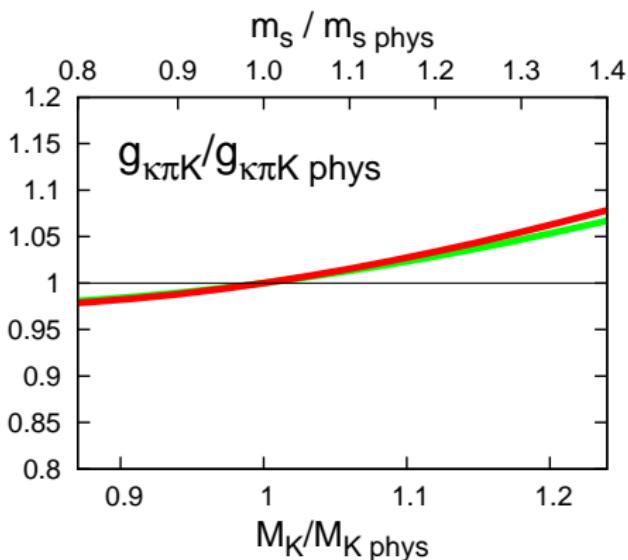
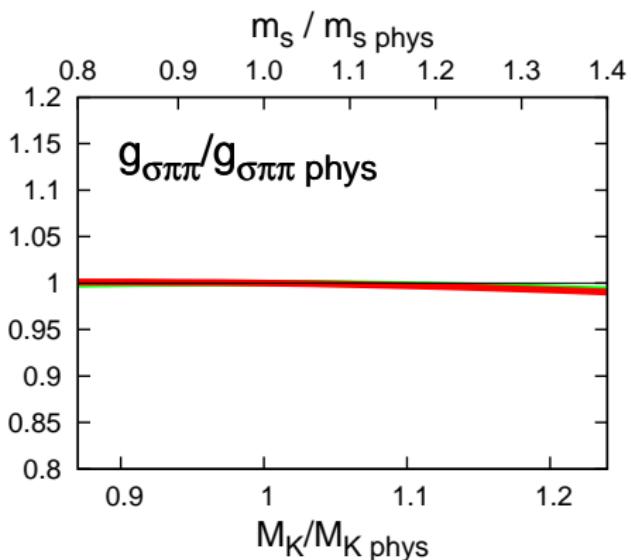
m_s dependence - Light scalar mesons - Mass & Width



m_s dependence - Light scalar mesons - Mass & Width



m_s dependence - Light scalar mesons - Coupling



Summary

Summary

We have presented some **preliminary** results for the phase shifts \hat{m} dependence:

Standard ChPT

- very soft dependence once threshold is "subtracted",
- somewhat stronger at two loops (specially for $I=2 J=2$),
- fair agreement with $I=2 J=0$ phase shifts (despite too high M_π).

Unitarized ChPT

- similar results to one loop, but up to the resonance region,
- work in progress in two loops.

Summary

Chiral extrapolation of the parameters of the σ ($f_0(600)$), $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing the quark masses \hat{m} and m_s .

Vector mesons:

- vector resonances mass grows slower than M_π ,
- coupling to two mesons almost independent of m_q ,
- KSFR is well satisfied for different quark masses.

Scalar mesons:

- very different behaviour from vector mesons: two branches,
- σ and κ show different quantitative but similar qualitative behaviour,
- coupling to two mesons shows stronger m_q dependence.